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A linear transformation between two vector spaces V and W is a map $T : V \to W$ such that the following hold:

1. $T(\mathbf{v}_1 + \mathbf{v}_2) = T(\mathbf{v}_1) + T(\mathbf{v}_2)$ for any vectors $\mathbf{v}_1$ and $\mathbf{v}_2$ in V, and
2. $T(\alpha \mathbf{v}) = \alpha T(\mathbf{v})$ for any scalar α.

The main example of a linear transformation is given by matrix multiplication. Given an $n \times m$ matrix A, define $T(\mathbf{v}) = \mathsf{A}\,\mathbf{v}$, where $\mathbf{v}$ is written as a column vector (with m coordinates). For example, consider

$$\mathsf{A} = \begin{bmatrix} 0 & 1 \\ -2 & 2 \\ 1 & 0 \end{bmatrix},$$

then T is a linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$, defined by

$$T(x, y) = (y, -2x + 2y, x).$$

Exercise.

Is each of the following transformation a linear transformation?

1. The transformation T defined by $T(x_1, x_2, x_3) = (x_1, 0, x_3)$
2. The transformation T defined by $T(x_1, x_2, x_3) = (1, x_2, x_3)$
3. The transformation T defined by $T(x_1, x_2) = (2x_1 - 3x_2, x_1 + 4, 5x_2)$
4. The transformation T defined by $T(x_1, x_2) = (4x_1 - 2x_2, 3|x_2|)$
5. The transformation T defined by $T(x_1, x_2, x_3, x_4) = 3x_1 + 4x_3 - 2x_4$
6. The transformation T defined by $T(x_1, x_2, x_3) = (x_1, x_2, -x_3)$