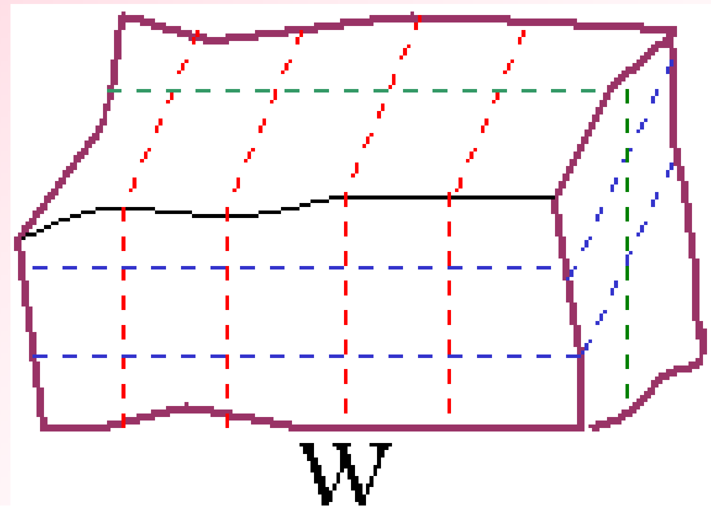
Website: bestmathtutor.ca Phone: 778-882-4636

Triple Integral

$$\iiint_W f(x,y,z)\,dV = \lim_{n\to+\infty}\sum_{i=1}^{n} f(x_i,y_i,z_i).\Delta V_i$$



Properties

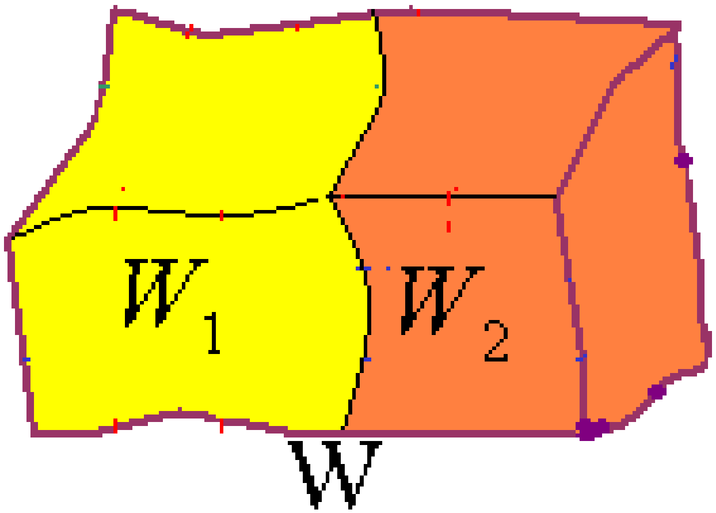
1) $$\iiint_W kf(x,y,z)\,dV = k\iiint_W f(x,y,z)\,dV$$

2) $$\iiint_W (f(x,y,z) \pm g(x,y,z))\,dV = \iiint_W f(x,y,z)\,dV \pm \iiint_W g(x,y,z)\,dV$$

3) $$f(x,y,z) \geq 0, \forall (x,y,z) \in W \Rightarrow \iiint_W f(x,y,z)\,dV \geq 0$$

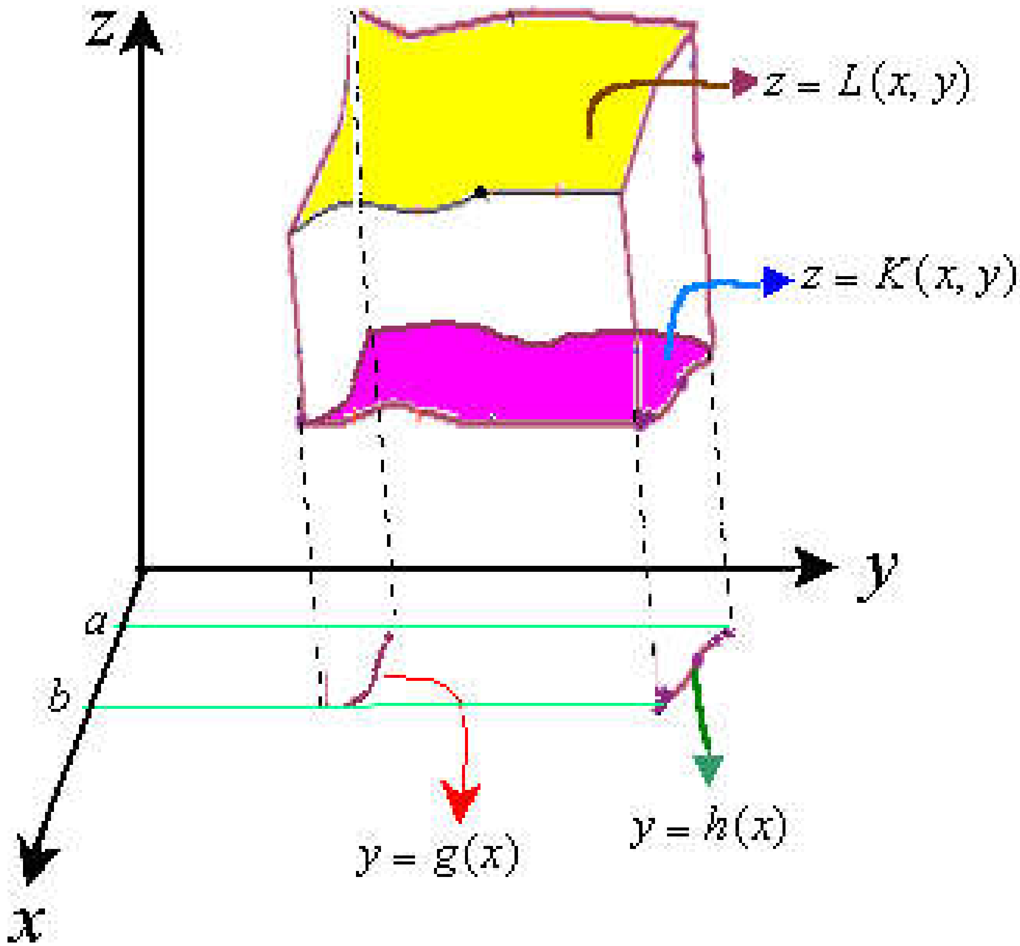
4) $f(x,y,z) \geq g(x,y,z), \forall (x,y,z) \in W \Rightarrow \iiint\limits_{W} f(x,y,z)\,dV \geq \iiint\limits_{W} g(x,y,z)\,dV$

5)



$$\iiint\limits_{W} f(x,y,z)\,dV = \iiint\limits_{W_1} f(x,y,z)\,dV + \iiint\limits_{W_2} f(x,y,z)\,dV$$

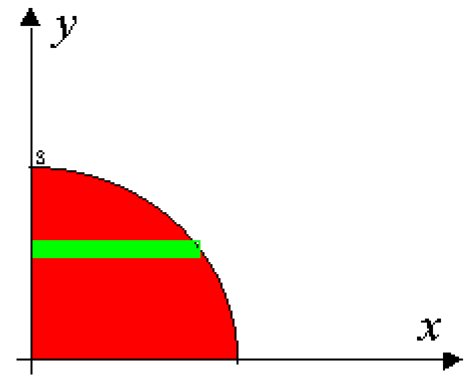
6)



$$\iiint_W f(x,y,z)\,dV = \int_a^b \int_{g(x)}^{h(x)} \int_{K(x,y)}^{L(x,y)} f(x,y,z)\,dz\,dy\,dx$$

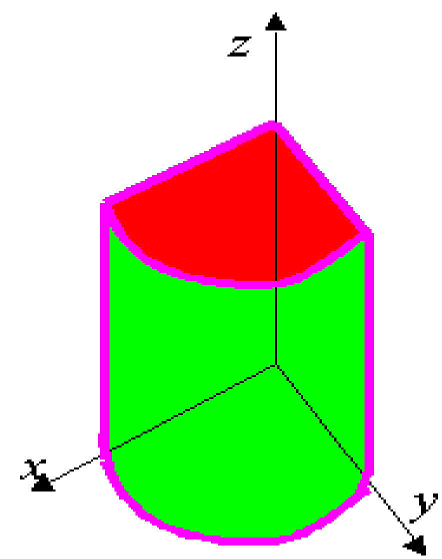
Exercise. Find

$$I = \int_0^4 \int_0^3 \int_0^{\sqrt{9-x^2}} \sqrt{9-y^2}\, dy\,dx\,dz$$



Solution.

$$\Rightarrow I = \int_0^3 \int_0^{\sqrt{9-y^2}} \int_0^4 \sqrt{9-y^2}\, dz\,dx\,dy$$



$$= \int_0^3 \int_0^{\sqrt{9-y^2}} 4\sqrt{9-y^2}\, dx\,dy = \int_0^3 4(9-y^2)\,dy$$

$$= 36y - \frac{4}{3}y^3 \Big|_0^3 = 72$$