MOCK MIDTERM MATH 101

Need More Help in Math?

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Question 1. Short Answer Questions. Evaluate:

(a) $\int_1^2 \frac{d}{dx}(\frac{sin(\frac{\pi}{2}x)}{x})dx$ **(b)** $\frac{d}{dx}\int_1^2(\frac{sin(\frac{\pi}{2}x)}{x})dx$ **(c)** The equation of tangent line to $f(x) = \int_{tan(x^3)-1}^{3^x} \frac{t}{\sqrt{t^4+1}}dt$ at $x = 0$.

(d) $\int_0^{\pi} cos(x).e^{-sin^2 x}dx$ **(e)** $\int_0^2 \sqrt{4-x^2}dx$ **(f)** $\lim_{x\to\infty}(\sqrt[3]{\frac{16\times 1}{n^4}} + \sqrt[3]{\frac{16\times 2}{n^4}} + \cdots + \sqrt[3]{\frac{16\times n}{n^4}})$

Solution. (a) By FTC 2: $= \frac{sin(\frac{\pi}{2}x)}{x}]_1^2 = \frac{sin(\pi)}{2} - \frac{sin(\frac{\pi}{2})}{1} = -1$.

(b) It is zero, because $\int_1^2(\frac{sin(\frac{\pi}{2}x)}{x})dx$ is a constant number.

(c) $f(0) = \int_{-1}^1 \frac{t}{\sqrt{t^4+1}}dt = 0$, because $y = \frac{t}{\sqrt{t^4+1}}$ is an odd function. By FTC 1: $f'(x) = (3^x)'.(\frac{3^x}{\sqrt{(3^x)^4+1}}) - (tan(x^3)-1)'.(\frac{tan(x^3)-1}{\sqrt{(tan(x^3)-1)^4+1}}) = 3^x.(ln3).(\frac{3^x}{\sqrt{3^{4x}+1}}) - sec^2(x^3).3x^2.(\frac{tan(x^3)-1}{\sqrt{(tan(x^3)-1)^4+1}})$; thus $f'(0) = \frac{ln3}{\sqrt{2}}$. So the tangent line is

$y - 0 = \frac{ln3}{\sqrt{2}}(x-0)$; that is $y = \frac{ln3}{\sqrt{2}}x$.

(d) Use substitution $sin(x) = u$, then $cos(x)dx = du$, $x = 0 \Rightarrow u = sin(0) = 0$,

$x = \pi \Rightarrow u = sin(\pi) = 0$, so it is $\int_0^0 e^{-u^2}du = 0$.

(e) It is the area of the region bounded by the semicircle $y = \sqrt{4-x^2}$ and x-axis and y-axis in Quadrant 1, so it is $\pi.(2)^2/4 = \pi$.

(f) $= \lim_{x\to\infty}\frac{1}{n}(\sqrt[3]{\frac{16\times 1}{n}} + \sqrt[3]{\frac{16\times 2}{n}} + \cdots + \sqrt[3]{\frac{16\times n}{n}}) = \int_0^1 \sqrt[3]{16x} = \sqrt[3]{16}\times\frac{3}{4}x^{\frac{4}{3}}]_0^1 = \frac{3\sqrt[3]{2}}{2}$.

Question 2. Find the area of the region bounded by the parabola $y = x^2 + 4$ and the lines $y = 4x$ and $y = -12x - 16$.

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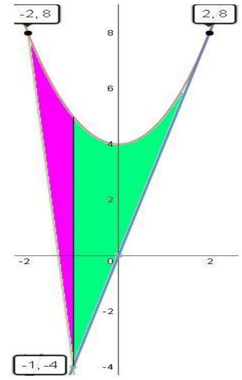
Solution. Area$=\int_{-2}^{-1}(x^2+4-(-12x-16))dx$

$$+\int_{-1}^{2}(x^2+4-(4x))dx=$$

$$(x^3/3+12x^2/2+20x)]_{-2}^{-1}+(x^3/3+4x-4x^2/2)]_{-1}^{2}$$

$$=40/3.$$



Question 3. Evaluate the volume of the solid obtained by rotating the region bounded by the curve $y=x^2$ and the line $y=3x$ about the line **(a)** $y=-1$ **(b)** $x=-2$. (Do not calculate the integrals)

Solution. (a) $V=\pi\int_0^3((3x-(-1))^2-(x^2-(-1))^2)dx$ **(b)** $V=\pi\int_0^3((\sqrt{y}-(-2))^2-(y/3-(-2))^2)dx$.

Question 4. A spherical tank of radius $4m$ is 1/4 full of an oil of density 1600 kg/m^3. Find the work required to pump the oil out of a spout $1m$ above the top of the tank.

Solution. $r_i^2=4^2-(x-5)^2=(16-(x^2-10x+25))=-x^2+10x-9$

Area of base $=\pi r_i^2=\pi(-x^2+10x-9)$ $V_i=$ Area of base $\times\Delta x=\pi(-x^2+10x-9)\Delta x$

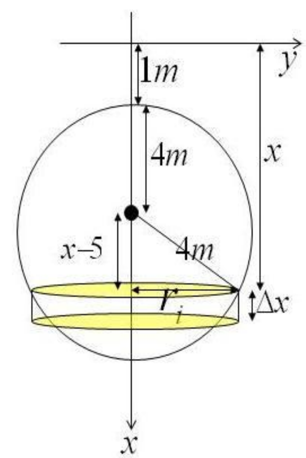
$m_i=V_i\times$ density $=1600\pi(-x^2+10x-9)\Delta x$

$f_i=m_i\times g=9.8\times1600\pi(-x^2+10x-9)\Delta x$

$w_i=f_i\times x=9.8\times1600\pi(-x^2+10x-9)x\,\Delta x$

$W=\int_7^9 9.8\times1600\pi(-x^2+10x-9)x\,dx=9.8\times1600\pi\int_7^9(-x^3+10x^2-9x)dx$

$=9.8\times1600\pi(-x^4/4+10x^3/3-9x^2/2)]_7^9=1609810\pi$ J



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