$$y = x^3\left(\ln\left(x^5 - 2\sqrt{1+3x^2-5x^7}\right)\right)^4 \qquad y' = ?$$

$$y' = 3x^2\left(\ln\left(x^5 - 2\sqrt{1+3x^2-5x^7}\right)\right)^4 +$$

$$4x^3\left(\ln\left(x^5 - 2\sqrt{1+3x^2-5x^7}\right)\right)^3 \times$$

$$\frac{1}{x^5 - 2\sqrt{1+3x^2-5x^7}} \times \left(5x^4 - \frac{6x-35x^6}{\sqrt{1+3x^2-5x^7}}\right)$$

$$\lim_{x\to 0^+}\frac{\ln(1+x)}{x}=?$$

$$1-\frac{1}{x}\le \ln x\le x-1 \Rightarrow \frac{x}{x+1}\le \ln(x+1)\le x$$

$$\xrightarrow{\times\frac{1}{x}} \frac{1}{1+x}\le \frac{\ln(x+1)}{x}\le 1$$

$$\lim_{x\to 0^+}\frac{1}{1+x}=\lim_{x\to 0^+} 1=1$$

$$\xrightarrow{\text{Squeeze Theorem}} \lim_{x\to 0^+}\frac{\ln(x+1)}{x}=1 \qquad (1)$$

$$\lim_{x \to 0^-} \frac{\ln(x+1)}{x} \qquad 1 - \frac{1}{x} \le \ln x \le x - 1 \implies$$

$$\frac{x}{x+1} \le \ln(x+1) \le x \xrightarrow{\times \frac{1}{x}} \frac{1}{1+x} \ge \frac{\ln(1+x)}{x} \ge 1$$

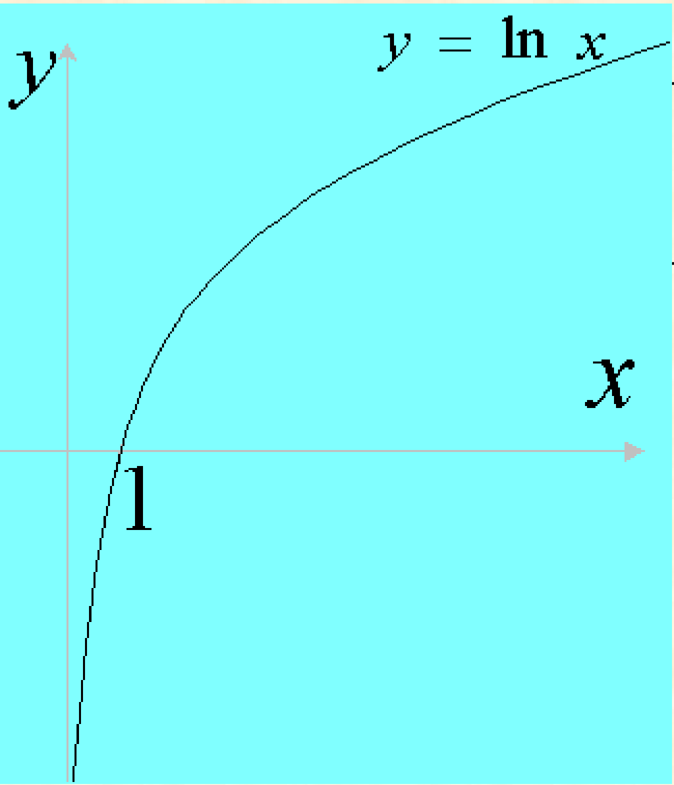
$$\lim_{x \to 0^-} \frac{1}{1+x} = \lim_{x \to 0^-} 1 = 1$$

$$\xrightarrow{\text{Squeeze Theorem}} \lim_{x \to 0^-} \frac{\ln(x+1)}{x} = 1 \qquad (2)$$

$$(1)\ ,\ (2) \implies \lim_{x \to 0} \frac{\ln(x+1)}{x} = 1$$

Exercise. Graph the fucntion: $y = \ln x$

$$y' = \frac{1}{x} \qquad \forall x > 0 \Rightarrow y' > 0 \qquad y'' = -\frac{1}{x^2} < 0$$



x	0		1		$+\infty$
y'			$+$		
y''			$-$		
y	$-\infty$	$\nearrow \cap$	0	$\nearrow \cap$	$+\infty$

$$y = x^5 \cdot e^{\frac{5x^3}{\sqrt{4x-5x^4}}} \qquad y' = ?$$

$$y' = 5x^4 \cdot e^{\frac{5x^3}{\sqrt{4x-5x^4}}} + x^5 \left(e^{\frac{5x^3}{\sqrt{4x-5x^4}}} \right) \times$$

$$\left(\frac{15x^2\sqrt{4x-5x^4} - 5x^3\left(\frac{4-20x^3}{2\sqrt{4x-5x^4}} \right)}{4x-5x^4} \right)$$

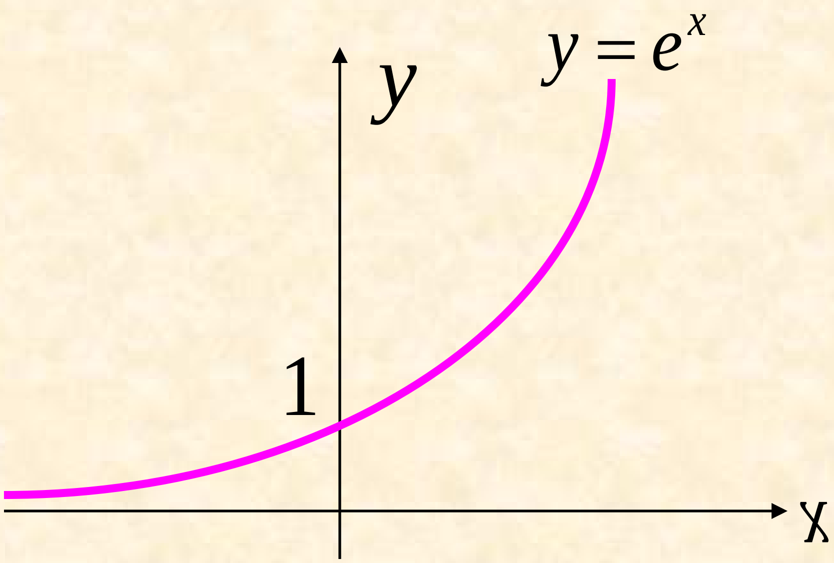
Exercise. Graph the fucntion: $y = e^x$

$$y' = e^x > 0 \quad , \quad y'' = e^x > 0$$

$$x \to +\infty \Rightarrow y \to +\infty$$

$$x \to -\infty \Rightarrow y \to 0^+$$

x	$-\infty$	0	$+\infty$
y'		$+$	
y''		$+$	
y	0	1	$+\infty$



Exercise. Prove that: $e^x \geq x^e \quad \forall x > 0$

$$e^x \geq x^e \quad \Leftrightarrow \ln e^x \geq \ln x^e \Leftrightarrow x \geq e \ln x$$

Define: $f(x) = x - e\ln x$

$$f(x) = x - e\ln x$$

$$f'(x) = 1 - \frac{e}{x} = 0 \Rightarrow$$

$$f'(x) = \frac{x-e}{x} = 0 \Rightarrow x = e$$

x	0		e		$+\infty$
y'		$-$	0	$+$	
y		$\searrow$	0 (min)	$\nearrow$	

$$f(x) \ge f(e) \Rightarrow x - e\ln x \ge 0$$

$$\Rightarrow x \ge e\ln x \Rightarrow e^x \ge e^{e\ln x} \Rightarrow e^x \ge x^e$$

(1) $y = \log_5 \left(\log(x + \csc^{-1}(x^9))^3\right)$

$$y' = ?$$

$$y' = \frac{1}{\ln 5 \times \log(x + \csc^{-1}(x^9))^3} \times \frac{1}{\ln 10 \times (x + \csc^{-1}(x^9))^3} \times$$

$$\times 3(x + \csc^{-1}(x^9))^2 \times \left(1 - \frac{9x^8}{|x^9|\sqrt{x^{18} - 1}}\right)$$

(2) $y = \log_{\csc^3 x} \sec^2(x^3)$

$$y' = ?$$

$$y = \frac{\ln \sec^2(x^3)}{\ln \csc^3 x}$$

$$y' = \frac{(\ln \csc^3 x)\left(\dfrac{1}{\sec^2(x^3)} \times \left(6x^2 \sec^2(x^3) \times \tan(x^3) \right)\right)}{(\ln \csc^3 x)^2} +$$

$$+ \frac{(\ln \sec^2(x^3))\left(\dfrac{3}{\csc^3 x} \times \csc^3 x \times \cot x\right)}{(\ln \csc^3 x)^2}$$

$$y = x^{\sin x} + 3(\sin x)^{(\cos x)^{(\tan x)}} \quad 0 < x < \frac{\pi}{2}$$

$$z = x^{\sin x} \Rightarrow \ln z = \ln x^{\sin x}$$

$$\Rightarrow \ln z = \sin x \cdot \ln x$$

Derivative $\longrightarrow$ $$\frac{z'}{z} = \cos x \cdot \ln x + \frac{\sin x}{x}$$

$$\Rightarrow \quad z' = x^{\sin x}\left(\cos x \cdot \ln x + \frac{\sin x}{x}\right)$$

$$u = \sin^{\cos^{\tan x} x} x \;\Rightarrow\; \ln u = \ln \sin^{\cos^{\tan x} x} x =$$

$$= \cos^{\tan x} x \cdot \ln \sin x$$

$$\xrightarrow{\text{مشتق}} \frac{u'}{u} = \left(\cos^{\tan x} x\right)' \cdot \ln \sin x + \left(\cos^{\tan x} x\right) \cdot \cot x$$

$$\Rightarrow u' = u \times (\ldots)$$

$$w = \cos^{\tan x} x \;\Rightarrow\; \ln w = (\tan x)\ln \cos x$$

$$\Rightarrow\; \frac{w'}{w} = (\sec^2 x)\ln\,\cos x + \tan x\left(\frac{-\sin x}{\cos x}\right)$$

$$\Rightarrow\; w' = w \times (\ldots)$$

$$\Rightarrow\; y' = z' + 3u'$$

$$(1)\qquad \lim_{x\to 0}(1+ax)^{\frac{b}{x}} \to 1^{\infty}$$

$$\Rightarrow\ y=\lim_{x\to 0}(1+ax)^{\frac{b}{x}}\ \Rightarrow\ \ln y=\ln\ \lim_{x\to 0}(1+ax)^{\frac{b}{x}}$$

$$\xlongequal[\text{}]{\text{پیوسته}\quad \ln x}\ \lim_{x\to 0}\ \ln(1+ax)^{\frac{b}{x}}=\lim_{x\to 0}\frac{b}{x}\ln(1+ax)$$

$$=b\lim_{x\to 0}\frac{\ln(1+ax)}{x}\ \xlongequal{Hopital}\ b\lim_{x\to 0}\frac{\frac{a}{1+ax}}{1}=ab$$

$$\Rightarrow\ \ln y=ab\ \Rightarrow\ y=e^{ab}$$

(2) $$\lim_{x\to 0^+} x^{\int_0^x \frac{\sin t}{t}dt} \to 0^0$$

$$y = \lim_{x\to 0^+} x^{\int_0^x \frac{\sin t}{t}dt}$$

$$\Rightarrow \ln y = \ln \lim_{x\to 0^+} x^{\int_0^x \frac{\sin t}{t}dt}$$ *Ln Continious*

$$\lim_{x\to 0^+} \ln x^{\int_0^x \frac{\sin t}{t}dt} = \lim_{x\to 0^+} \int_0^x \frac{\sin t}{t}dt \times \ln x =$$

$$\lim_{x\to 0^+} \frac{\ln x}{\dfrac{1}{\int_0^x \frac{\sin t}{t}\,dt}} \overset{Hopital}{=\!=\!=} \frac{\dfrac{1}{x}}{-\dfrac{\dfrac{\sin x}{x}}{\left(\int_0^x \frac{\sin t}{t}\,dt\right)^2}} =$$

$$= \lim_{x\to 0^+} \frac{\left(\int_0^x \frac{\sin t}{t}\,dt\right)^2}{-\sin x}$$

$$\underset{\overline{\overline{}}}{\textit{Hopital}} \lim_{x\to 0^+} \frac{-2\left(\int_0^x \frac{\sin t}{t}\,dt\right)\left(\frac{\sin x}{x}\right)}{\cos x}$$

$$= -2\lim_{x\to 0^+} \frac{\sin x}{x} \times \underbrace{\lim_{x\to 0^+} \frac{\int_0^x \frac{\sin t}{t}}{\cos x}}_{0} = 0 \Rightarrow \ln y = 0 \Rightarrow y = 1$$

$$(3)\qquad \lim_{x\to 0^+} x^{\int_0^x \sin t^2 dt} \to 0^0$$

$$y=\lim_{x\to 0^+} x^{\int_0^x \sin t^2 dt} \Rightarrow \ln y = \ln \lim_{x\to 0^+} x^{\int_0^x \sin t^2 dt}$$

$$\xlongequal[\text{}]{\text{پیوسته } \ln} \lim_{x\to 0^+} \ln x^{\int_0^x \sin t^2 dt} =$$

$$\lim_{x\to 0^+} \int_0^x \sin t^2 dt \times \ln x = \lim_{x\to 0^+} \frac{\ln x}{\dfrac{1}{\int_0^x \sin t^2 dt}}$$

$$\lim_{x\to 0^+}\frac{\ln x}{\dfrac{1}{\int_0^x \sin t^2\,dt}} \overset{Hopital}{=\!=\!=} \lim_{x\to 0^+}\frac{\dfrac{1}{x}}{\dfrac{-\sin x^2}{\left(\int_0^x \sin t^2\,dt\right)^2}} =$$

$$= \lim_{x\to 0^+}\frac{\left(\int_0^x \sin t^2\,dt\right)^2}{-x\sin x^2} = \lim_{x\to 0^+}\frac{2\left(\int_0^x \sin t^2\,dt\right)\sin x^2}{-\sin x^2 - 2x^2\cos x^2}$$

$$= \lim_{x\to 0^+}\frac{2\left(\int_0^x \sin t^2\,dt\right)\dfrac{\sin x^2}{x^2}}{-\dfrac{\sin x^2}{x^2} - 2\cos x^2} = \lim_{x\to 0^+}\frac{2\times 0\times 1}{-1-2\times 1} = 0$$

$$\Rightarrow \ln y = 0 \Rightarrow y = 1$$

$$y = x^5 \sinh^3\left(\cosh^5(x^4) + \tanh^3(x^7 - 2x^5 - 1)\right) + \operatorname{sec}h^4(x^2)$$

$$y' = ?$$

$$y' = 5x^4 \sinh^3\left(\cosh^5(x^4) + \tanh^3(x^7 - 2x^5 - 1)\right) +$$

$$+ 3x^5 \sinh^2\left(\cosh^5(x^4) + \tanh^3(x^7 - 2x^5 - 1)\right) \times$$

$$\times \cosh\left(\cosh^5(x^4) + \tanh^3(x^7 - 2x^5 - 1)\right) \times$$

$$\left(5\cosh^4(x^4).\sinh(x^4).4x^3 + 3\tanh^2(x^7 - 2x^5 - 1).\right.$$

$$\left.\operatorname{sec}h^2(x^7 - 2x^5 - 1).(7x^6 - 10x^4)\right)$$

$$+ 4\operatorname{sec}h^3(x^2).(-\operatorname{sec}h(x^2).\tanh(x^2)).2x$$