Find $\lim_{x\to 3} \dfrac{x^2-5x+6}{x^2-4x+3}$.

Solution. $x \to 3 \Rightarrow Limit: \dfrac{9-15+6}{9-12+3} \quad \dfrac{0}{0}$; So factor the numerator and the denumenator

$$Limit = \lim_{x\to 3}\frac{(x-3)(x-2)}{(x-3)(x-1)} = \lim_{x\to 3}\frac{(x-2)}{(x-1)} = \frac{1}{2}$$

Evaluate the following limit:

$$\lim_{h\to 0}\frac{\sqrt{9+h}-3}{h}.$$

Solution. $h \to 0 \Rightarrow Limit: \dfrac{\sqrt{9}-3}{0} \quad \dfrac{0}{0}$; So

multiply the numerator and the denumenator by the conjugate of the numerator :

$$Limit = \lim_{h\to 0}\frac{(\sqrt{9+h}-3)(\sqrt{9+h}+3)}{h(\sqrt{9+h}+3)} = \lim_{h\to 0}\frac{(\sqrt{9+h}^{2}-9)}{h(\sqrt{9+h}+3)} == \lim_{h\to 0}\frac{h}{h(\sqrt{9+h}+3)}$$

$$= \lim_{h\to 0}\frac{1}{(\sqrt{9+h}+3)} = \frac{1}{\sqrt{9}+3} = \frac{1}{6}$$

Suppose

$$f(x) = \begin{cases} x^2 - 5x & \text{if } x \le -1, \\ ax^3 - 7 & \text{if } x > -1. \end{cases}$$

Determine all values of the constant a for which $\lim_{x\to -1} f(x)$ exists, and state the value of the limit, if possible.

Solution. $\lim_{\substack{x\to -1^{+} \\ x>-1}} f(x) = \lim_{x\to -1^{+}} (ax^3 - 7) = a(-1)^3 - 7 = -a - 7;$

$$\lim_{\substack{x\to -1^{-} \\ x<-1}} f(x) = \lim_{x\to -1^{+}} (x^2 - 5x) = (-1)^2 - 5(-1) = 6;$$

S0 we must have: $-a - 7 = 6 \Rightarrow a = -13$ *and* $\text{Limit} = \text{Left Limit} = 6$

If $\lim_{x\to 1} f(x) = 8$ and $\lim_{x\to 1} g(x) = 3$, then find $\lim_{x\to 1} \sqrt[3]{f(x)g(x)+3}$.

Solution. $\lim_{x\to 1} f(x).g(x)+3 = \lim_{x\to 1} f(x).\lim_{x\to 1} g(x) + \lim_{x\to 1} 3 = 8\times 3+3 = 27$

So $Limit = \sqrt[3]{27} = 3.$

Determine the value of the constant a for which the function

$$f(x) = \begin{cases} \dfrac{x^2+3x+2}{x+1} & \text{if } x \neq -1, \\ a & \text{if } x = -1 \end{cases}$$

is continuous at $x = -1$.

Solution. For the values $x \neq -1$, the fuction is the rational function $y = \frac{x^2+3x+2}{x+1}$

and the denumenator is not zero, so the fuction is continious at any x ≠ -1.

Now for $x = -1$: $f(-1) = a$, $\lim_{\substack{x\to -1 \\ x\neq -1}} f(x) = \lim_{x\to -1} \frac{x^2+3x+2}{x+1} = \lim_{x\to -1} \frac{(x+1)(x+2)}{(x+1)}$

$= \lim_{x\to -1} \frac{(x+2)}{1} = -1+2 = 1;$ *and* $f(-1) = \lim_{x\to -1} f(x) \Rightarrow a = 1.$

Question. Find the tangent line to the curve $y = Sin\, x$ at the point $x = \frac{\pi}{6}$.

$$m = Slope = \lim_{x \to \frac{\pi}{6}} \frac{f(x) - f(\frac{\pi}{6})}{x - \frac{\pi}{6}} = \lim_{x \to \frac{\pi}{6}} \frac{\sin x - \sin\frac{\pi}{6}}{x - \frac{\pi}{6}} = \lim_{x \to \frac{\pi}{6}} \frac{\sin x - \frac{1}{2}}{x - \frac{\pi}{6}}$$

$$x - \frac{\pi}{6} = t \Rightarrow x = t + \frac{\pi}{6}, \quad x \to \frac{\pi}{6} \Rightarrow t \to 0$$

$$\Rightarrow m = \lim_{t \to 0} \frac{\sin(t + \frac{\pi}{6}) - \frac{1}{2}}{t} = \lim_{t \to 0} \frac{\sin t.\cos\frac{\pi}{6} + \cos t.\sin\frac{\pi}{6} - \frac{1}{2}}{t}$$

$$= \lim_{t \to 0} \frac{\frac{\sqrt{3}}{2}\sin t + \frac{1}{2}\cos t - \frac{1}{2}}{t} = \frac{\sqrt{3}}{2}\lim_{t \to 0}\frac{\sin t}{t} + \frac{1}{2}\lim_{t \to 0}\frac{\cos t - 1}{t} = \frac{\sqrt{3}}{2}$$

Tangent Line: $y - f(a) = m(x - a)$

$$y - \sin\frac{\pi}{6} = \frac{\sqrt{3}}{2}(x - \frac{\pi}{6})$$

$$\Rightarrow y - \frac{1}{2} = \frac{\sqrt{3}}{2}(x - \frac{\pi}{6})$$